CONVERT POLAR EQUATION TO RECTANGULAR

eg. $r = \sin 2\theta$

FROM TRIG: $\sin 2\theta = 2\sin\theta\cos\theta$

METHOD 1: ① REWRITE IN TERMS OF 2 BASIC TRIG FUNCTIONS OF θ ONLY
IE. $\sin\theta$, $\cos\theta$

$$r = 2\sin\theta\cos\theta$$

② MULTIPLY BY r's SO ALL $\sin\theta$, $\cos\theta$ APPEAR AS $r\sin\theta$, $r\cos\theta$

$$r(r) = (2\sin\theta\cos\theta)rr$$

$$r^3 = 2(r\sin\theta)(r\cos\theta)$$

③ LET $r\sin\theta = y$, $r\cos\theta = x$

$$r^3 = 2yx$$

④ REWRITE r AS $\sqrt{x^2+y^2}$

$$\sqrt{x^2+y^2}^{\,3} = 2xy$$

⑤ SIMPLIFY (NO $\sqrt{\ }$, FRACTIONS, NEGATIVE/FRACTIONAL EXPONENTS)

$$\left(\left(\sqrt{x^2+y^2}\right)^3\right)^2 = (2xy)^2$$

$$\boxed{(x^2+y^2)^3 = 4x^2y^2}$$

$\cancel{\tfrac{1}{2}} \cdot 3 \cdot \cancel{2}$

$$\left(\left(x^2+y^2\right)^{\frac{1}{2}}\right)^{3}\Big)^{2}$$

10.7 WEBSITE HANDOUT

METHOD 2: $r = \sin 2\theta$

① REWRITE IN TERMS OF TRIG FUNCTIONS OF θ ONLY

IE. $\sin\theta, \cos\theta, \tan\theta, \sec\theta, \csc\theta, \cot\theta$

$$r = 2\sin\theta\cos\theta$$

② REWRITE TRIG FUNCTIONS IN TERMS OF x, y, r

$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

$$\csc\theta = \frac{r}{y} \quad \sec\theta = \frac{r}{x} \quad \cot\theta = \frac{x}{y}$$

$$r = 2\frac{y}{r}\frac{x}{r}$$

$$r^2(r) = \left(\frac{2xy}{r^2}\right)r^2$$

$$r^3 = 2xy$$

(*) IMMEDIATELY MULTIPLY ALL EQUATIONS WITH FRACTIONS BY LCD TO ELIMINATE DENOMINATORS

③ REWRITE r AS $(x^2+y^2)^{\frac{1}{2}}$

$$\left((x^2+y^2)^{\frac{1}{2}}\right)^3 = 2xy$$

$$(x^2+y^2)^{\frac{3}{2}} = 2xy$$

④ SIMPLIFY

$$\left((x^2+y^2)^{\frac{3}{2}}\right)^2 = (2xy)^2$$

$$\boxed{(x^2+y^2)^3 = 4x^2y^2}$$

CONVERT $r = \frac{2}{1+3\sin\theta}$ TO RECTANGULAR

METHOD 2:

1

2 $r(1+3\sin\theta) = 2$

$r(1+3\cdot\frac{y}{r}) = 2$

$r + 3y = 2$

3 $(x^2+y^2)^{\frac{1}{2}} + 3y = 2$

4 $(x^2+y^2)^{\frac{1}{2}} = 2-3y$

$((x^2+y^2)^{\frac{1}{2}})^2 = (2-3y)^2$

$x^2+y^2 = (2-3y)^2$

QUADRATIC IN X,Y

$x^2+y^2 = 4-12y+9y^2$

$x^2-8y^2+12y-4 = 0$

OPPOSITE SIGNS

→ HYPERBOLA

10.8 POLAR GRAPHS

NEED 16 POINTS IF PLOTTING POINTS ONLY

$r = \sin\theta$

FOR GRAPHING

$\sqrt{2} \approx 1.4$

$\sqrt{3} \approx 1.8$

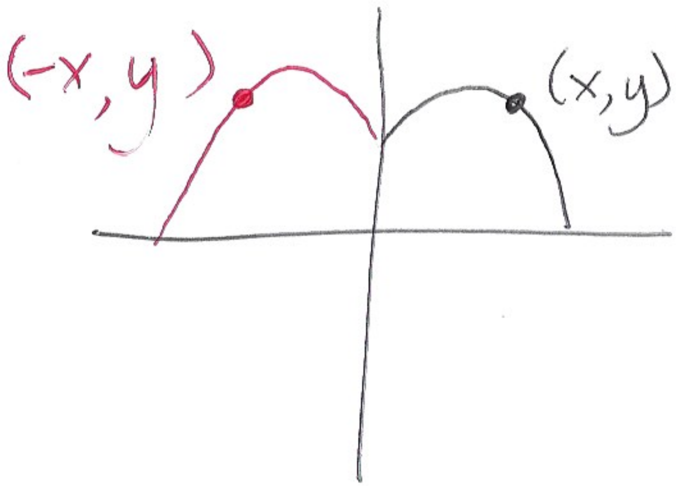
θ	$r = \sin\theta$	(r, θ)
0	$\sin 0 = 0$	$(0, 0)$
$\frac{\pi}{6}$	$\sin\frac{\pi}{6} = \frac{1}{2}$	$\left(\frac{1}{2}, \frac{\pi}{6}\right) = \left(0.5, \frac{\pi}{6}\right)$
$\frac{\pi}{4}$	$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right) \approx \left(0.7, \frac{\pi}{4}\right)$
$\frac{\pi}{3}$	$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3}\right) \approx \left(0.9, \frac{\pi}{3}\right)$
$\frac{\pi}{2}$	$\sin\frac{\pi}{2} = 1$	$\left(1, \frac{\pi}{2}\right)$
$\frac{2\pi}{3}$		
$\frac{3\pi}{4}$		
$\frac{5\pi}{6}$		
π		
$\frac{7\pi}{6}$		
$\frac{5\pi}{4}$ ...		

$\left.\begin{matrix} 0 \\ \frac{\pi}{6} \\ \frac{\pi}{4} \\ \frac{\pi}{3} \end{matrix}\right\}$ ALL IN Q_1

$\left.\begin{matrix} \frac{\pi}{2} \\ \frac{2\pi}{3} \\ \frac{3\pi}{4} \\ \frac{5\pi}{6} \end{matrix}\right\}$ IN Q_2

$\left.\begin{matrix} \pi \\ \frac{7\pi}{6} \\ \frac{5\pi}{4} \end{matrix}\right\}$ IN Q_3

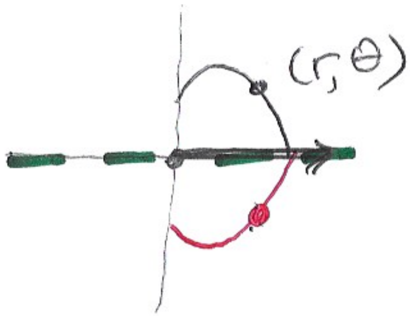
REMINDER: IF A RELATION IN x, y
HAS A GRAPH THAT IS SYMMETRIC OVER y-AXIS



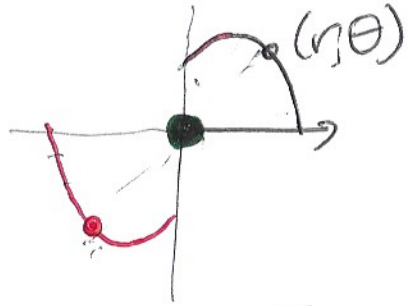
TO TEST IF A RELATION IN x, y
IS SYM OVER y-AXIS:

REPLACE (x, y) WITH $(-x, y)$
IN RELATION
AND IF IT CAN BE SIMPLIFIED
BACK TO THE ORIGINAL
RELATION, THE GRAPH IS
SYM OVER y-AXIS
IF IT CAN'T BE SIMPLIFIED BACK,
THE GRAPH IS NOT SYM OVER y-AX

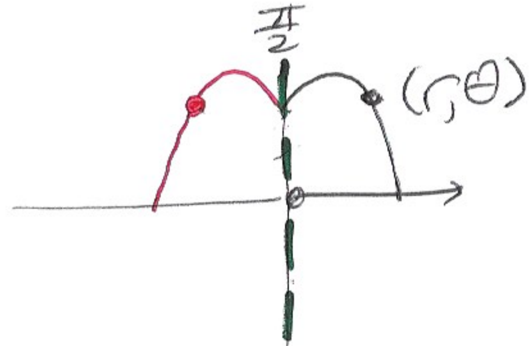
3 TYPES OF SYMMETRY IN POLAR:



OVER POLAR AXIS



OVER POLE



OVER $\theta = \frac{\pi}{2}$